Minimising total energy requirements in amplified links by optimising amplifier spacing.

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Abstract: We investigate the energy optimization (minimization) for amplified links. We show that using the using a well-established analytic nonlinear signal-to-noise ratio noise model that for a simple amplifier model there are very clear, fiber independent, amplifier gains which minimize the total energy requirement. With a generalized amplifier model we establish the spacing for the optimum power per bit as well as the nonlinear limited optimum power. An amplifier spacing corresponding to 13 dB gain is shown to be a suitable compromise for practical amplifiers operating at the optimum nonlinear power.

OCIS codes: (060.4510) Optical communications; (060.1660) Coherent communications.

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1. Introduction

Optical links are generally designed to carry the maximum data capacity for the longest distance at lowest cost. Key design parameters available to planners are the signal powers and amplifier spacing. The specific performance achievable will in addition depend on parameters such as fibre loss, amplifier noise figure, nonlinear coefficient and modulation formats. Fibre nonlinearity limits the Shannon limited capacity and simple models of this ultimate limit have recently been derived [1, 2]. Advances in modulation format and FEC have allowed this limit to be approached in practical systems [3].

The optimum signal powers which allow the nonlinear limit for capacity to be reached are key results of these works. The majority of previous work sought to maximise the system performance and optimised the signal power only in respect to performance [4–7]. We have recently shown [8] that contrastingly if the total optical power required for an entire system is considered the amplifier spacing which minimises this power may be identified, allowing systems to be designed with the lowest energy requirements. The underlying assumption is that the electrical power needed to operate an amplified optical link is directly related to the total optical power employed by the sum of the amplifiers. Previous work on optimum power [9] was restricted to the linear regime and showed that for a lossless amplifier the minimum power per bit is obtained with distributed amplification i.e. zero gain amplifiers. In this paper we review the results of [8] showing the impact of including nonlinearity on the optimum amplifier spacing. We further extend this work to provide a more generalised amplifier power consumption model and verify that a finite optimum amplifier spacing always exists, and provide concise prescriptions to provide bounds for this optimum.

The results of the work here will be important both from a simple operating cost perspective as well as providing a significant contribution to the task of reducing energy in communication systems. In particular we will show that there is a very simple prescription for the optimum amplifier gain (and span length) to obtain the best power performance overall which is independent of the system parameters with the exception of loss.

2. Initial analytical results

In the following we will use the nonlinear noise model described in [2] which treats the impact of nonlinearity as a noise term proportional to the cube of the signal power and has been shown to be reliable for uncompensated coherently detected optical links. Here we consider a chain of N amplifiers with equal gain and express the signal-to-noise ratio (SNR) including the cubic nonlinear term. This SNR can be used to derive the channel capacity limit. The key results of this paper do not depend on the details of the modulation format employed and apply for any system where the nonlinear 'noise' term is proportional to the signal power cubed. We write the SNR as:

$$SNR = \frac{P_S / B_{amp}}{N(G-1)\sigma_{ASE} + NL_{eff}\sigma_{NL} P_S^3 / B_{amp}^3} \qquad (1)$$

Where P_S is the sum of the individual channel powers (W), B_{amp} represents the overall bandwidth of the system (Hz), G the amplifier gain (in the absence of excess loss G is given by $e^{\alpha L}$, where L is the span length and α the loss coefficient), L_{eff} the nonlinear effective length of each span (m) and σ_{ASE}, σ_{NL} are considered length independent parameters determining the strength of the amplified spontaneous (W/Hz) and nonlinear noise (./W^2/s^2/m) respectively. The key results detailed in this paper are independent of the actual values of σ_{ASE}, σ_{NL} however in order to calculate achievable information spectral densities we have used a common form of the Gaussian Noise Model, assuming system bandwidths exceeding a few THz, a continuous spectrum and the incorporation of polarization effects into the nonlinear coefficient γ (./W/m), where these parameters are closely approximated by [2]:

$$\sigma_{ase} = n_{sp} h\upsilon, \quad \sigma_{NL} = \frac{\gamma^2 c}{\lambda^2 D} \log\left(\frac{\pi B_{amp}^2 \lambda^2 D}{\alpha c}\right) \qquad (2)$$

where n_{sp} is the spontaneous emission factor, ν the frequency (Hz), D the delay dispersion (s/m^2) and α the loss coefficient (./m). However scaling these parameters to account for reduced bandwidth or spectral guard bands does not influence the final conclusion of this work. Equation (1) has a very simple optimum which occurs when the nonlinear term equals one half of the ASE term (the first term in the dominator of Eq. (1)). A simple analytic form for this optimum power has been derived [1, 2].

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$$\frac{P_{opt}}{B_{amp}} = \sqrt[3]{\frac{(G-1)\sigma_{ASE}}{2L_{eff}\sigma_{NL}}} \cong \sqrt[3]{\frac{\alpha G \sigma_{ASE}}{2\sigma_{NL}}} \quad (3)$$

where the approximation is valid for large G. In order to investigate the total power used in a system consisting of a chain of N (identical) amplifiers we now rewrite Eq. (1) in terms of the total (optical) power P_T (at fixed span length L)

$$SNR_T = \frac{NP_T / B_{amp}}{N^3 (G-1)\sigma_{ASE} + L_{eff}\sigma_{NL} P_T^3 / B_{amp}^3} \quad (4)$$

$$P_T = NP_S \quad (5)$$

We can consider the total power added, $P_{add\text{-}T}$, by the amplifiers by considering internal loss coefficients Λ_{in} and Λ_{out} at the input and output to the amplifiers respectively. In this case the generalized version of Eq. (5) is:

$$P_{add-T} = NP_s\left(\frac{1}{\Lambda_{out}} - \frac{\Lambda_{in}}{G}\right) \quad (6)$$

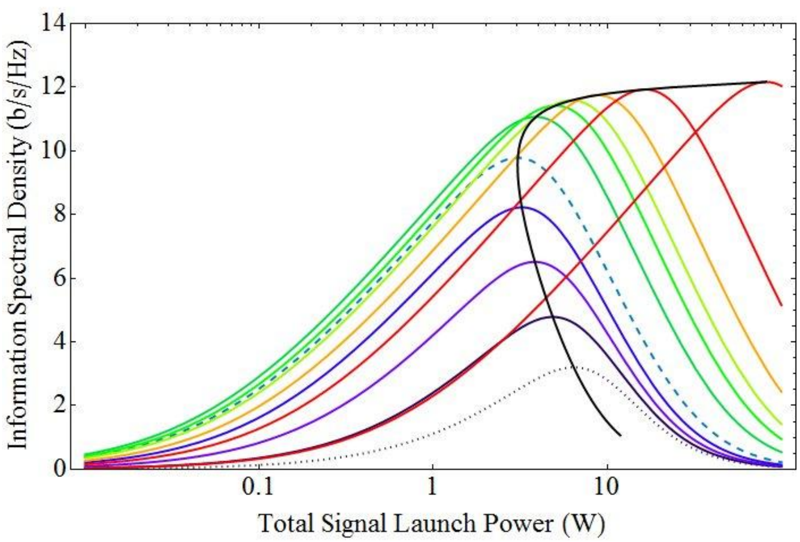

Fig. 1. Spectral density against total optical launch power for a fixed 3000 km system with a 5.THz bandwidth, channels spaced at 50 GHz, a fiber loss of 0.2 dB/km, nonlinear coefficient of 1.4 /W/km, dispersion coefficient of 20 ps/nm/km and an amplifier noise figure of 4.7 dB. For a range of amplifier spans; 180 (dark blue dotted line), 150, 120, 90, 60 (blue dashed line), 30, 20, 15, 10, 5, and 1 (solid red line) km, and showing the locus of ISD for operation at the nonlinear threshold as the amplifier spacing is varied (thick black line).

In the following, to fully illustrate the implications of total power optimization, we will consider an example dual polarization system with system and fiber parameters (unless otherwise stated): dispersion 20 ps/nm/km, loss 0.046 /km (0.2 dB/km), nonlinearity 1.4/W/km (taking into account polarization effects), system length 3000 km. But the results may be generalized to all current calculations of nonlinear capacity [2].

We will first consider the total power as the sum of the amplifier output powers (equivalent to considering $\Lambda_{in} = 1$ and $\Lambda_{out} = 1$ in Eq. (6)). In this limit we assume the relevant optical power which scales the required electrical power is the output power of the amplifiers. Whilst this is a limit which is accurate for significant amplifier gains it is not accurate for

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very small amplifier gain (i.e. as the linear gain approaches unity) but we will show that this model leads to very simple, interesting and useful results.

Figure 1 shows the result of using Eq. (4) to generate the information spectral density (ISD) plotted against the sum of the amplifier output powers for a range of amplifier spacing. The black line in the plot follows the peak of the curves, often referred to as the nonlinear threshold of the system and which may be readily calculated using Eqs. (3) and (4). This line has a clear turning point which shows that the total optical power employed by the system has a minimum. The figure illustrates that as the amplifier spacing is decreased the peak ISD also increases whilst the total power decreases down to a spacing of about 60 km after which the power increases. Alternatively rather than operate the system at the nonlinear threshold an amplifier spacing may be selected to minimize the required total launch power for a specified target ISD. For a wide range of ISD values, the minimum required total power under this strategy is indicated the green line which corresponds to an amplifier spacing of much less than 60 km but shows only marginal improvement over the ISD obtained with the ~60 km spacing. Note also that these graphs illustrate that after the turning point of the black line the line is almost flat with a log power dependence indicating that a very large increase in power is needed to obtain the small increase in capacity offered by very short spans.

3. General analytical results

In the below we derive some important and simple analytical results under the assumption of large amplifier loss considering only the optical output powers of the amplifiers in the chain. We will consider two approaches to calculate the minimum total signal power. The first corresponds to the conventional model of system operation, namely to operate at the nonlinear limited threshold of the system. Mathematically this is equivalent to taking the optical power given by Eq. (3) and finding the span length for which this is minimized by solving $\partial P_T / \partial L = 0$. After some simple algebraic manipulation, this can be shown to give the following expression for the amplifier span

$$L_a = \frac{3}{\alpha} \tag{7}$$

This is a remarkably simple expression which is independent of the system parameters and the majority of the fiber parameters (dispersion, nonlinearity etc.). Indeed in terms of gain, this gives a universal optimum gain of 13 dB (or expressed linearly; e^3) to ensure the minimum power consumption when the system is operated at the nonlinear threshold, as given by Eq. (3). This is not the global minimum power which, of course, will depend on the desired SNR. But this gain is at the point where increasing the amplifier spacing not only reduces the SNR but also increases the total required system power i.e. this is the turning point illustrated in Fig. 1. Thus it is desirable for both performance and power reasons not to exceed this amplifier span length which is 65 km for a routinely encountered loss of 0.2 dB/km

We will turn our attention to the more global case and optimize Eq. (4) with respect to amplifier gain. As will become clear later the minimum required total signal power is obtained through the solution of $\partial P_T / \partial L = 0$ directly (without first insisting on operation at the nonlinear threshold of Eq. (3)). There is no direct analytical solution to the equation which results from rearranging Eq. (4) and taking the differential, however, by differentiating Eq. (4) with respect to length, and asserting the conditions that firstly this differential should be zero and secondly by selecting the solution which also requires that $\partial P_T / \partial L = 0$ this process may be shown to correspond to finding the solution of:

$$2 + e^{\alpha L}\left(\alpha L - 2\right) + e^{-\alpha L}\frac{\sigma_{NL}}{\sigma_{ASE}}\frac{1}{N^3}\left(\frac{\alpha L + e^{+\alpha \cdot L} - 1}{\alpha}\right)P_T^3 = 0 \tag{8}$$

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Again, there is no exact general solution to Eq. (8) but it does have a very simple asymptotic solution when we take the limit towards zero power (and of course zero SNR).

$$L_b = \frac{2 + W\left(-2/e^2\right)}{\alpha} \tag{9}$$

where W(.) represents the Lambert W function, the inverse function for $f(W) = We^W$. For span lengths below L_b the minimum required total power only increases. There is an obvious similarity between Eqs. (7) and (9) in that both indicate a universal optimum span length only dependent on loss, and in consequence a universal amplifier gain. Ignoring the correction factor W(.) the difference in optimum linear gains is simply e, and including it the difference increases to 6 dB. Note that in practice, the impact of nonlinearity is to reduce L_b slightly.

4. Exemplar graphical results

We will now plot Eq. (4) and results (5) to (9) on a single graph shown in Fig. 2.

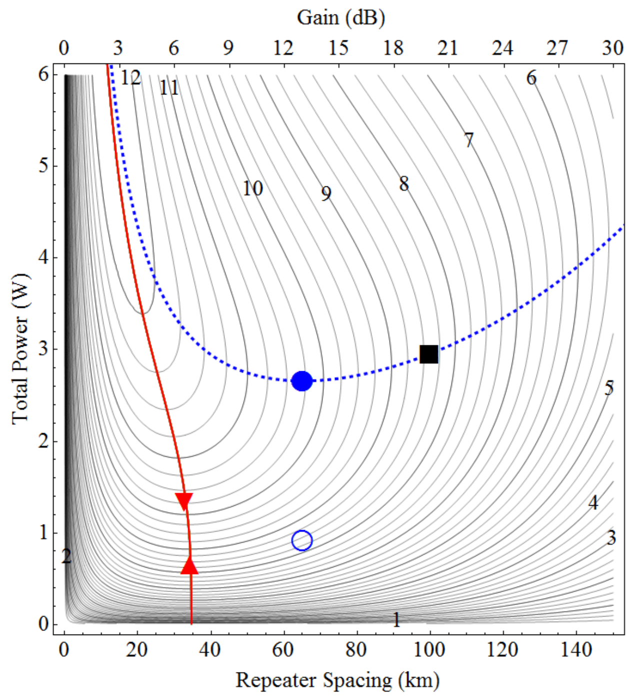

Fig. 2. Achievable ISD (contours) as a function of total amplifier output optical power vs amplifier spacing with dispersion 20 ps/nm/km, loss 0.046 /km (0.2 dB/km), nonlinearity 1.4/W/km and a system length of 3000 km. Blue dotted line is the optimum nonlinear capacity for a given amplifier span. The red line is the minimum total power for a specific capacity.

The figure shows contours of nonlinear Shannon capacity for a polarization multiplexed system (2 $\mathrm{Log}_2(1 + \mathrm{SNR}_T)$) in the (amplifier spacing, total power) space typically used by system designers (also shown is the amplifier gain which is useful for the fiber independent interpretation). The blue dotted line is the optimum capacity against amplifier span. It runs along the SNR contour ridge and diverges as the amplifier spacing tends to zero. This shows that, although the vanishingly small amplifier spacing will give the maximum performance, the overall power requirement tends to infinity.

The solid red line in Fig. 2 highlights the minimum total power for a given SNR i.e. the solution to Eq. (8). This power is always below that of the blue dotted line and intercepts the x-axis at a spacing given by Eq. (9). A simple illustration will show how to use Fig. 2 and what potential energy savings are available. The black square is the optimum possible

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capacity with 100 km span length. The open blue circle is the point with the same capacity, but at the spacing given by Eq. (5). There is a substantial reduction in power (69%) whilst retaining the same performance. Thus the desired capacity could have been achieved with less than half the power if the amplifier spacing had been reduced from 100 to 65 km. A reduction is almost as large (46%) going from 80 to 65 km can also been seen. For the 100 km case a further power reduction of ~28% can be achieved by continuing on the contour to intercept the red line which for the optimum value to match the 100 km capacity (34.5 km) is almost indistinguishable from the asymptotic value in Eq. (9) i.e. 34.6 km.

5. Extension to generalized amplifier model

All of the above is based on a model where the sum of the output powers of the amplifiers was taken as the appropriate quantity for optimizing (minimizing) the required system power. Now we extend the work to include the model given by Eq. (6) for the power *added*, P_{add-T}, by the amplifier. This is an important consideration for amplifiers operating close to the optima discussed here, since the input power represents small but observable fraction of the output power, and the coupling losses a small but observable fraction of the total loss. Now we will repeat the calculations above but varying Λ_{in} and Λ_{out} (note these are linear losses with zero loss represented by 1 and infinite loss, 0). Using Eq. (6) for the power leads only to a simple analytical results for operation at the nonlinear threshold, and for the minimum total power solution we will have to content ourselves with simply providing graphical results in what follows. By combining Eq. (6) and Eq. (4) the deliverable signal to noise ratio becomes:

$$SNR_{add-T} = \frac{NP_T / B_{amp}}{N^3 \left(G-1\right) \dfrac{\sigma_{ASE}}{\Lambda_{in}} + L_{eff} \sigma_{NL} \left(\dfrac{G \cdot \Lambda_{out} P_T / B_{amp}}{G - \Lambda_{out} \Lambda_{in}} \right)^3} \tag{10}$$

Figure 3 shows a contour plots for two cases; the ideal case, with no insertion losses, and a more practical case with around 1.5 dB loss at the amplifier input and 1.5 dB loss at the output i.e. Λ_{in} = 0.7 and Λ_{out} = 0.7 . The left hand (3a) plot, calculated for negligible input and output losses, corresponds to the case considered in [9]. However if coupling losses are neglected the lowest overall output power strategy suggests that distributed amplification represents the minimum power consumption, consistent to [9], although we find that an optimum amplifier spacing for a system operated at the nonlinear threshold (46.7 km) still remains. The first conclusion is clearly unphysical, as common sense suggests that there must be some energy penalty for the infinite number of pump couplers associated with infinitesimally short amplifier spacing. The optimum associated with the nonlinear threshold occurs due to the increasing impact of nonlinearity for amplifiers spaced below the fiber effective length. Including practical insertion losses resolves the unphysical issues for the shortest amplifier spacing, and has the consequence of transforming the achievable ISD plot to one very similar to Fig. 2 showing essentially the same features. Widening the range of amplifier spacings for which Eq. (6) is a good approximation and we find that the optimum amplifier spacing for systems operated at the nonlinear threshold is 59.3 km, close to the total output power result. The asymptotic minimum power corresponds to an amplifier spacing of 28.3 km for this choice of input and output losses.

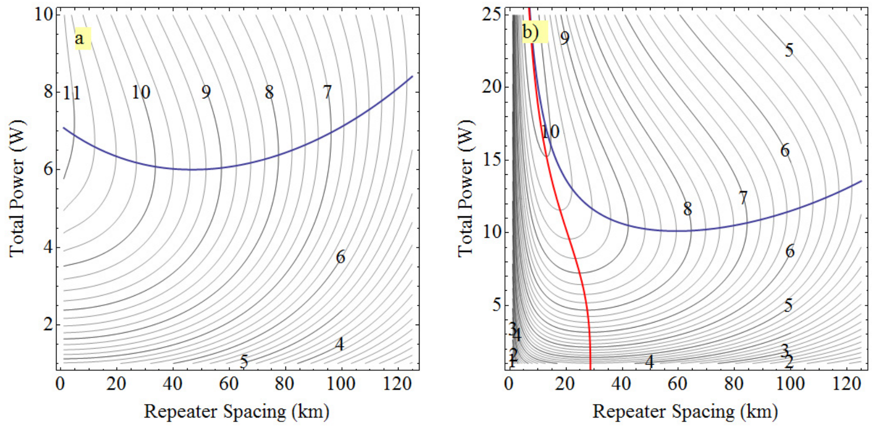

Fig. 3. Achievable ISD (contours) as a function of total added optical power and amplifier spacing with a dispersion 20 ps/nm/km, loss 0.046 /km (0.2 dB/km), nonlinearity 1.1/W/km and a system length of 6,000 km. The insertion loss of each amplifier is 0 dB (a) and 3 dB (b) equally distributed between the amplifier input and output.

As explained above we have two 'optimum operating scenarios'. The first is to operate with the amplifier spacing which has the minimum total system power where the amplifiers are adjusted to their appropriate nonlinear thresholds. The second is simply the minimum power per bit for the system. Figure 4 shows the amplifier spacing for these two limits as a function of Λ_{in} (in dB) for various values of Λ_{out} (in dB).

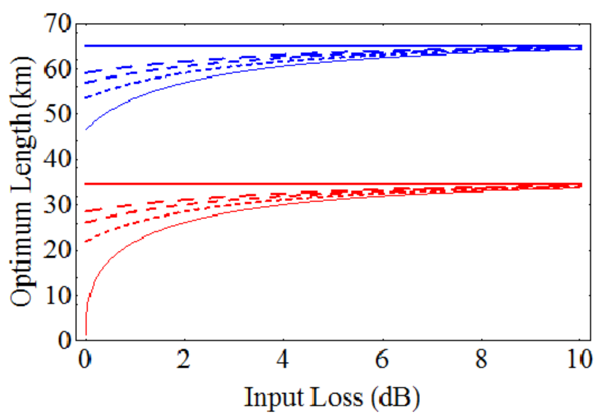

Fig. 4. Optimum amplifier spacing versus insertion loss of all components at the amplifier input for a fiber with 0.2 dB/km loss, assuming operation at the nonlinear threshold (blue curves) and the minimum possible added signal power (red curves). Lines represent; thin solid – 0 dB output loss, short dashed – 1 dB, medium dash – 2 dB and long dashed - 3 dB output losses. Thick solid lines represent analytical approximations based on total output power (Eqs. (7) and (9) respectively). Note that the amplifier population inversion (n_{sp}) rather than effective black-box noise figure is held constant in this plot.

The first point to note is that for the lossless amplifier $\Lambda_{in} = \Lambda_{out} = 1$ (0dB) the minimum spacing for the optimum power per bit is zero which is the result given in [9]. However for the nonlinear optimum even for the perfect amplifier the optimum spacing is finite and equal to 46.7 km for a fiber loss of 0.2 dB/km. In both cases, as the insertion losses increase, the optimum amplifier spacing asymptotically approaches the simple values expressed in Eqs. (7) and (9), suggesting that the simple 13 dB (or 63 km) amplifier is a good target for a nonlinearly limited system with a practical amplifier. Indeed, we can find no scenario where it is energy efficient to utilize a net amplifier gain in excess of the value predicted by Eq. (7).

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The importance of the nonlinear threshold limit is that having installed a system with a chosen length it is almost inevitable that this would be used up to its limit defined by the nonlinearity. Therefore this limit would be used regardless of any other intentions.

It is interesting to note that whilst the optimum amplifier spacing for operation at the nonlinear threshold (minimum total power) for conventional fiber is 65 (35) km, for ultra-low loss fiber with a loss coefficient of 0.149 dB/km [10], this increases to ~90 (48) km, in line with current network deployments. Of course, optimization based on the functional power requirements as described here tells only part of the story, with overheads associated with control and management also contributing to the total power consumption [9], and hence optimization as described here. We note however that since their first introduction in the 1990s [11], a wide range of power overheads have been associated with optical amplifiers ranging from zero [11] to several Watts per amplifier. If such excess power exists, the total electrical power consumption is given by:

$$P_{elect} = N\left(\frac{P_s}{\eta}\left(\frac{1}{\Lambda_{out}} - \frac{\Lambda_{in}}{G}\right) + P_{overhead}\right) \tag{11}$$

where $P_{overhead}$ is the power overhead per amplifier and η the overall power conversion efficiency of the amplifier including cooling, insertion losses and all appropriate conversion efficiencies (typically η ~0.1). The equivalent SNR ratio equation is straightforward to calculate, but as the range of possible overheads is immense we leave the details of this exercise to the reader, noting however that the result will be to push the optimum towards slightly longer amplifier spacing, and that both the importance of this correction and the excess power consumption associated with control and management may be readily judged from Eq. (11).

6. Conclusions

The simple model presented here produces some very clear and widely applicable results on the power minimization of amplified fiber links. In particular we have shown that there are high potential savings in energy demands for optically amplified links which can be obtained by the appropriate selection the amplifier spacing (and associated output power). This spacing is generally significantly less than the current designs. We observe that a span length requiring a 13 dB gain will give, universally, most of the benefit. But even shorter amplifier spans will give further benefit.

Of course the savings identified here are for power or equivalently operational costs. Increased amplifier count would contribute to the capital costs. The 13 dB optimum gain may seem low compared with currently operating land based systems, however, for advanced fibers with losses of 0.16 dB/km the spacing returns to the standard 80 km. Note that in this respect, if new fibers are to be deployed, loss is the only parameter which has any impact on the optimum amplifier spacing and for this any many other reasons, loss remains the key fiber parameter on which to concentrate.

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